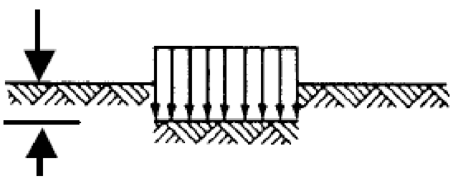
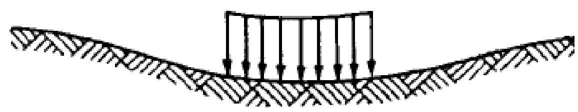
Modeling of Subgrade

Elastic foundation models: advantage vs disadvantage

It is critical for subgrade models to reflect realistic soil behavior in mechanistic pavement design. The figure below shows two more commonly used models for elastic foundation (subgrade), namely Winkler model and Elastic Solid model.



Winkler Model



Elastic Solid Model

The Winkler model simplifies all the soil strata to a bed of springs that has a linear spring constant or k-value. While it is probably the most widely used due to its simplicity and acceptance by the engineering community, it has the disadvantage of ignoring the shear deformation (load transfer between the springs) due to soil cohesion and friction. Furthermore, it assumes a linear relationship between applied pressure and deflection. As a result, Winkler model will result in the unrealistic deflection profile as shown above and it also tends to underestimate the support of soil to the superstructure. Elastic Solid model, on the other hand, overestimates the soil cohesion and friction by modeling the discrete soil particles as a solid medium. In summary, neither model is a true representation of the real soil behavior. Due to its simplicity, the Winkler model is used in both analytical solutions such as Westergaard equations and finite element analysis such as ISLAB. The use of Elastic Solid model is often limited to finite element analysis with advanced soil constitutive models to capture the nonlinear response.

Modeling subgrade in AASHTO Pavement ME

Thin Concrete Pavement (TCP) models pavement structures similar to the AASHTO Pavement ME design guide. As illustrated in the Figure below, in AASHTO Pavement ME all the layers are characterized by their stiffness (Young’s modulus or resilient modulus). In the structural analysis, all the layers below the top two layers are simplified into a bed of springs, which is somewhat a hybrid of Winkler model and Elastic Solid model rendering it capable of simulating a more realistic behavior between the two.

Static k-value VS Dynamic k-value VS Effective Dynamic k-value

It is also noteworthy that the stiffness of the bed of springs in AASHTO Pavement ME is termed as "effective dynamic k-value", which is "a computed value, not a direct input to the design procure". It is different from the static k-value typically used in foundation designs or previous pavement design procedures such as AASHTO 93.

The ASTM D1196 plate load test determines a static k-value that reflects the stiffness of the underlying bulb of soil that is within ~2 times of the plate size. The dynamic k-value is typically determined from Falling Weight Deflectometer (FWD) test, where a standard weight of 9 kips is dropped and the resultant profile of surface deflections is measured to backcalculate the dynamic k-value. It is often found that the dynamic k-value from FWD tests is approximately twice the static k-value from plate load test. Having discussed static k-value vs dynamic k-value, the "effective dynamic k-value" used in pavement modeling is again different in that it is not measured but computed. This is done by first computing the surface deflections due to loading the pavement layers of known moduli with a 9-kip FWD load in an elastic layer program JULEA and then backcalculating a dynamic k-value based on these computed deflections.

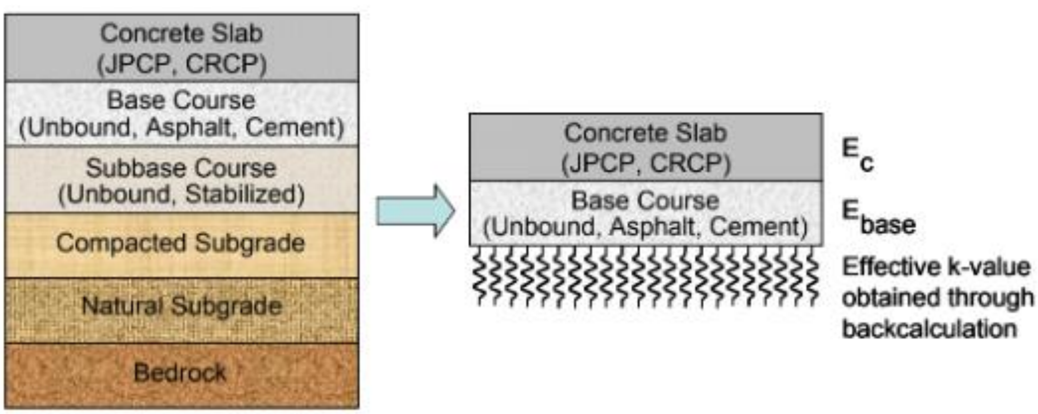


Figure 3.4.14. Structural model for rigid pavement structural response computations.

Details of how to determine the effective dynamic k-value in TCP is presented in the Appendix.

Appendix: Determining effective dynamic k-value in TCP

1. Deriving k-value from elastic layered system analysis

To determine effective dynamic k-value, the deflections due to a FWD load need to be computed. Deflection of a pavement is defined as the displacement that represents the structural response to the application of an external vertical load[1]. In reality, applying a load to the surface not only displaces the points under the load, but also displaces an area around the load, which is also known as the deflection bowl. Physically, deflection is indicative of the structural capacity of pavements, e.g. higher deflection potential indicates lower structural capacity and vice versa. This is also obvious to see from the definition of k-value below, i.e. the quotient between the pressure q (kg/cm^2) and the deflection Δ (cm) resulting from said pressure, where k-value or the stiffness of support is inversely related to deflection.

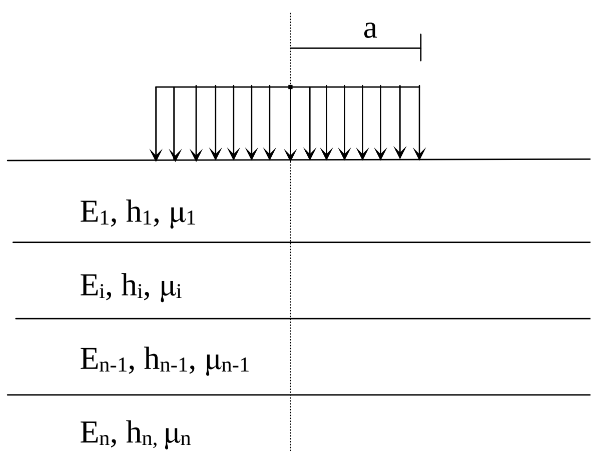
$$k = q / \Delta \qquad (kg / cm^3) \qquad (1)$$

Where:

k: Modulus of subgrade reaction (kg / cm^3)

q: applied pressure (kg / cm^2)

Δ: Deflection (cm)



Elastic multilayer systems

[1]Higuera, Carlos, the deflection basins in flexible pavement structures, Magazine Facultad de Ingenieria, UPTC, pp. 22

Where:

E_i = Modulus of elasticity of layer i

h_i = Thickness of layer i

μ_i = Poisson ratio of layer i

q = Contact pressure

a = radius of load

In this analysis, following Burmister's theory, an elastic multilayer system was assumed that has the following characteristic:

- There is a linear relationship between the stresses and the deformations.
- The material properties of each layer are homogeneous and isotropic.
- There is friction between two layers.
- Each layer has a finite thickness, except the lowest layer; Furthermore, all layers are infinite in horizontal directions.
- The material properties of each layer are defined by two main properties: modulus of elasticity (E) and Poisson's ratio (μ).
- A uniform pressure (q) is applied to the surface, in a circular area of radius a.

2. Calculating deflection

Numerous scientists, such as Boussinesq, Palmer, Barber and Odemark, have proposed various mathematical expressions for the calculation of surface displacement from the application of a uniform circular load in a structural system. The solution by Palmer and Barber is employed here. The total deflection at the center of the surface of the multilayer system is given by the following mathematical expression:

$$\Delta_0 = \frac{2qa\left(1-\mu^2\right)}{E_n}\left[\frac{\left(1-\frac{E_n}{\hat{E}}\right)}{\left[1+\left(\frac{h_1+h_2+\ldots+h_{n-1}}{a}\right)^2\left(\frac{\hat{E}}{E_n}\right)^{2/3}\right]^{1/2}}+\frac{E_n}{\hat{E}}\right] \quad (2)$$

where the equivalent modulus Ê is defined as below, and assuming the Poisson' ratio is the same for all the underlying layers i.e. $\mu_i = \mu_n$.

$$\hat{E} = E_1 \left[\frac{h_1 + h_2 \sqrt[3]{\frac{E_2}{E_1}} + h_3 \sqrt[3]{\frac{E_3}{E_1}} + \ldots + h_{n-1} \sqrt[3]{\frac{E_{n-1}}{E_1}}}{\sum_{i=1}^{n-1} h_i} \right]^3 \quad (3)$$

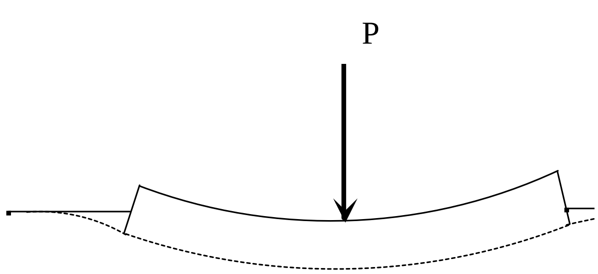
Replacing deflection in Equation (1) by Equation (2) will result in a k-value independent of the applied load, as shown in Equation (4).

$$k = \frac{1}{\left(\frac{2a(1-\mu^2)}{E_n} \left[\frac{\left(1 - \frac{E_n}{\hat{E}}\right)}{\left[1 + \left(\frac{h_1 + h_2 + \ldots + h_{n-1}}{a}\right)^2 \left(\frac{\hat{E}}{E_n}\right)^{2/3}\right]^{1/2}} + \frac{E_n}{\hat{E}} \right] \right)} \quad (4)$$

3. Correction for deflection

To more accurately capture soil behavior for pavement modeling, adjustments were made to correct deflection and to account for humidity.

The deflections used in the model correspond to vertical displacements on the surface when applying a uniform circular pressure q (kg/cm2). This unit pressure is equivalent to applying a load P on a flexible circular plate.

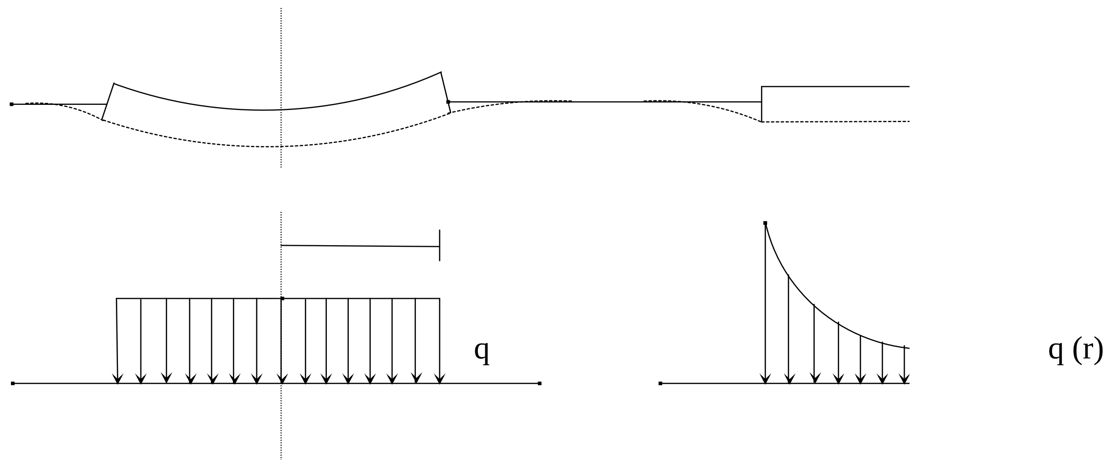


Model of a uniformly distributed load

where the load P is equivalent to:

$$P = \pi \cdot a^2 \cdot q \tag{5}$$

However, for either pavement or plate load test we need to model the deformation produced by a more rigid plate. An absolute rigid plate creates uniform deflection but different pressure under it. The difference between an absolutely flexible and an absolutely rigid plate can be seen in the following Figure.



Deflection and Pressure under - (a) Flexible plate and (b) Rigid plate

In Figure (b), to produce a uniform deflection the variable q(r) has been expressed by P. Ullidtz in 1987 as the following function.

$$q(r) = \frac{qa}{2(a^2 - r^2)^{0.5}} \tag{6}$$

Then, the load P is given by the integral of q(r) with respect to the radius of the load area.

$$P = \pi a \cdot \int_o^a \frac{qa}{2(a^2 - r^2)^{0.5}} dr \tag{7}$$

Or

$$P = \pi \cdot a^2 \cdot \left(\frac{q\pi}{4}\right) \tag{8}$$

Comparing Equations (5) and (8) it is noted that the pressure for a rigid plate is equivalent to 78.53% of the pressure of a uniformly distributed load on a flexible plate. As mentioned above, all the expressions proposed for the calculation of deflection are directly proportional to the applied pressure. Therefore, the deflection at the surface of a load plate is equivalent to 78.53% of the deflection at the center of a uniformly distributed load. Since having a lower deflection will result in greater structural capacity, the subgrade reaction or k-value should be increased by 27.32% (1 / 0.7853 = 1.2732), as shown in the equation below.

$$k = \frac{1.2732}{\left(\frac{2 \cdot a \cdot \left(1-\mu^2\right)}{E_n} \left[\frac{\left(1-\frac{E_n}{\hat{E}}\right)}{\left[1+\left(\frac{h_1+h_2+\ldots+h_{n-1}}{a}\right)^2 \left(\frac{\hat{E}}{E_n}\right)^{2/3}\right]^{1/2}} + \frac{E_n}{\hat{E}} \right] \right)} \tag{9}$$

Solving, finally we arrive to the formula that models a load plate test for a multilayer structural system:

$$k = \frac{0.0167087 \cdot E_n}{\left(1-\mu^2\right) \left[\frac{\left(1-\frac{E_n}{\hat{E}}\right)}{\left[1+\left(\frac{h_1+h_2+\ldots+h_{n-1}}{38.1}\right)^2 \left(\frac{\hat{E}}{E_n}\right)^{2/3}\right]^{1/2}} + \frac{E_n}{\hat{E}} \right]} \tag{10}$$

Where:

$$\hat{E} = E_1 \left[\frac{h_1 + h_2 \sqrt[3]{\frac{E_2}{E_1}} + h_3 \sqrt[3]{\frac{E_3}{E_1}} + \ldots + h_{n-1} \sqrt[3]{\frac{E_{n-1}}{E_1}}}{\sum_{i=1}^{n-1} h_i} \right]^3 \tag{11}$$

Ê = Equivalent Module

Ei = Modulus of Elasticity of layer i

hi = Layer thickness i

μ = Poisson ratio of layer n

4. Conclusion

In conclusion, k-value is defined differently depending on the design procedures. During testing, it varies with the size and application rate of the load among many other variables. It is not a fundamental parameter describing soil properties. During modeling, it is not the sole indicator of the reaction of subgrade to loading, which is also a function of the thickness of the concrete slab and the stress state of the underlying layers. In Mechanistic pavement modeling, such as TCP and AASHTO Pavement ME or seismic floor design, the effective dynamic k-value is not a direct input but a computed intermediate value.